Învățământ, Cercetare, Creație
Vol. 4 No. 1 - 2018

Golden Ratio in Sound Structures of Béla Bartók's Works

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Abstract: *In music, the golden ratio can appear in various ways, being found in the construction of instruments, in musical architectures or in the raw material of the music, the sound or rhythmic structures used. Thus, the forms of the golden section report, more precisely the numerical values of the Fibonacci sequence, can be noticed within the sound organizations by correlating these elements expressed numerically with corresponding elements specific to the art of sounds. Such a relationship can be made starting from the semitone content of the musical intervals whose joining, which respects the indicated proportions, can generate sound modes or structures that correspond to the musical golden ratio. Interesting sound structures are born in this way based on a special type of sound relationship, which opens a new perspective of harmony and reveals a special way of organizing proportions at the musical level.*

Key-words: *Golden Ratio; Fibonacci sequence; sound structures.*

Sound structures corresponding to the golden ratio

An interesting theory that points the relationship with the golden ratio is that of attunement, based on the frequency ratio between sounds is supporting the natural resonance and the fractions determined by Pythagoras. Thus, different frequency sequences of the Fibonacci sequences can be made, which translate into musical language in consonant intervals:

Ratio	Musical Interval	Stability
1 / 1	Perfect Unison	Perfect Consonant
2 / 1	Perfect Octave	Perfect Consonant
3 / 2	Perfect Fifth	Perfect Consonant
5 / 3	Major Sixth	Imperfect Consonant
8 / 5	Minor Sixth	Imperfect Consonant
13 / 8	Major sixth / minor sixth	Imperfect Consonant

Tab. 1 *Stability of music intervals*

H. E. Huntley argues in the *Divine Proportion: Study of the mathematical beauty* that from a musical point of view, the unison with the perfect octave are the most consonant intervals, and the expression of the golden section reported most suggestively among the musical sounds is given by the overtones who build a minor sixth, which reflects the 8/5 ratio.

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Starting from this idea, the sounds systems it can be re-evaluate or even the well-known sound structures and re-interpret them in terms of the golden ratio. For example, it is highlighted that the major chord is realized by a fitting of golden section reports since the component pitches correspond to the overtones (harmonics series) of the Fibonacci sequence (1, 2, 3, 5, 8).

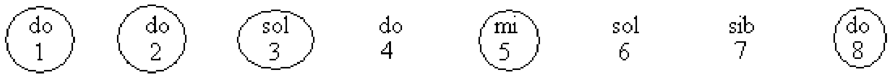

Fig. 1 *Overtones*

Sound structures built on the golden ratio in Béla Bartok's works

The musicologist Ernő Lendvai (1925-1993), known for his analysis of Béla Bartók's works, developed a wide-ranging analytical process in two directions, aiming the incidence of the golden report: on the one hand, the sound structures are investigated, and on the other hand the architectural form is followed. In the work *Béla Bartók: An analysis of his music*, E. Lendvai examines, among other things, some principles of structural construction based on golden section. The author believes that this study of proportions in Bartók's music can be extended to the sound system used, where intervals and the entire sound structure "*is based on the laws of golden section, especially on Fibonacci series"*.[2]

Based on the correspondence between the numerical value from the Fibonacci sequence and the content of the semitones from the musical intervals, the following connection can be established:

	Number of semitones	**Music interval**
1.	2 semitones	Major second
2.	3 semitones	Minor third
3.	5 semitones	Perfect fourth
4.	8 semitones	Minor sixth
5.	13 semitones	Augmented octave

Tab. 2. *Correspondence to musical interval*

In addition, the whole musical texture can also be imagined as a construction based on cells of 2, 3, 5, 8 and 13 unities whose subdivisions follow the proportions given by Fibonacci sequence (8 can be divided into 5 + 3, each of these elements being part of the same numerical sequence).

Fibonacci sequence							
1	2	3	5	8	13	21	34
(1)	(1+1)	(1+2)	(2+3)	(3+5)	(5+8)	(8+13)	(13+21)

Tab. 3 *Fibonacci sequence*

This idea can be seen at the end of the *Divertimento for String Orchestra* (1939) by B. Bartók, where the theme is presented in five hypostases, each one has a characteristic way of organizing.The development process envisages an evolution based on the golden ratio model

[2] Ernő Lendvai, *Béla Bartók: An Analysis of his Music*, London: Kahn and Averill, 1971 p. 35

melodic contour changes occur through intervals corresponding to the Fibonacci sequence (3 is replaced with 5).

In a detailed analysis of the value in the semitones of the musical intervals, Ernö Lendvai points out that the *Sonata for two pianos and percussion (*1937) by B. Bartok is built on the principle of golden ratio. Here, the leading motif (leitmotif) has 8 semitones, divided by the fundamental note (C) into two segments of 5 and 3 semitones. The principal theme contains 13 semitones divided by the fundamental (C) in 8 and 5 semitones. Then the first phrase of the secondary theme extends over 13 semitones from G to F#, and the second phrase has 21 semitones from B to D. Thus, all these important elements are built on the golden section: the leitmotif (8), the main theme (13), and the secondary theme (13 and 21). At the same time, this exposition provides a clear picture over the sound structure. The principal theme also has a diatonic "color" through the pentatonic harmony, a formula made by intervals of two and three semitones (2 + 3 + 2). In the middle segment of the principal theme is introduced an ostinato formula made by a structure which is containing two minor thirds and a perfect fifth in the middle, corresponding to the three and five semitone (3 + 5 + 3): C - E b – A b – B. The forth E b – A b is divided by F# in three and two semitones (3 + 2).

In Bartók's music we can see a certain preference for the parallel intervals whose value in halftones corresponds to this sequence: minor third (3), perfect fourth (5), minor sixth (8) and even major second (2). It seems that intervals like the major third (4) or the major sixth (9) appear rarely because they do not fit into the numerical system of the gold ratio, being practically incompatible with it. One can even notice the tendency to prohibit these intervals, in the same way that parallel fifths or octaves are forbidden in classical harmony. Lendvai mentions that Bartók prefers to use the minor third, appreciating his motivational role over the major third[3]. This is why, when Bartók uses the triad in a chromatic motion, he places the minor third above the root note and the major third under it, thus agreeing to the golden proportion (5 + 3; 3 + 5). Moreover, from the synthesis of the two intervals appears a typical bartókian chord, a characteristic element of its creation, a major-minor chord formed by a minor third - a perfect fourth - a minor third. This chord is often supplemented with a seventh (E – G – C – E b – B b).

Fig. 2 *Major – minor chord*

Thus, a sound organization is formed from a minor third overlay (3 semitones), welded in the middle by a major second (2 semitones). In Bartok's music this sound structure (3-3-3-2-3-3-3) is called *alpha chord*.

Ernő Lendvai believes that the principle of the golden ratio is not an external rule applicable to music, but even one of its most interesting internal laws. This is justified by one of the elementary sound systems, which can also be seen as pure (natural) expression of the golden ratio. In any anhemitonic diatonic pentatonic scale, for example, the sequence of steps generates intervals corresponding to the value values in the *Fibonacci sequence*: major second – 2, minor third – 3, perfect fourth – 5. It can be deduced from this the idea that the internal

3 Ernő Lendvai, *Béla Bartók: An Analysis of his Music,* p. 40

laws that govern this point of the construction of the scale, indicating a nucleus formed by the major second to which a minor third ascends and descends, corresponds fully to the proportions related to the golden ratio.

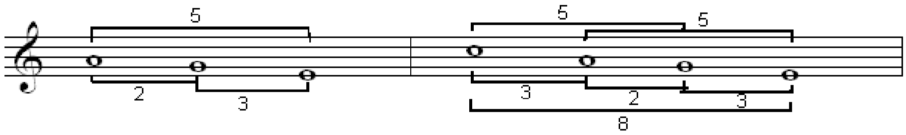

Fig. 3 *Music interval that corresponds to golden ratio*

In the same direction is also the opinion of Zoltan Kodály, who considers that the element generating pentatonic scale is the minor third, as it reflects basic tonal relationships, which represent the primary forms of musical expression much more convincing than the construction based on the perfect fifth succession.[4]

Expressions of pentatonic scale in B. Bartók's music can be found in various works. In the *Dance Suite* (1923), for example, the use of primary elements of the pentatonic scale shows a possible construction (of the sound material) based on golden section. The all movements show an interest for successive elements within Fibonacci series expressed in particular in each of the movement:

- First movement - evolves from a motif mostly built from major seconds (2 semitones);
- Second movement - has the minor third as a characteristic feature (3);
- Third movement - gathers together the previous elements, building a pentatonic scale with the following succession: major second, minor third, major second, minor third, major second (2 + 3 + 2 + 3 + 2). In addition, the harmonic support of this movement is based on forth successions that has 5 semitones.
- Fourth movements - presents a melodic profile that follows a model made by intervals such as minor sixth and perfect forth following the pattern by formulas such as 8 = 5 + 3 where 5 = 3 + 2.

Even when pentatonic structures are explicitly used, as in the song "I Have No One in the World" in the *Children's Choir and Women's Voice Collection*, highlights sometimes in the sound structure analysis a close connection of this diatonic system with the theory of the axial system and the alpha chord. From the pentatonic scale presented in the example below, two structures of the alpha (gamma and delta) chord are revealed.

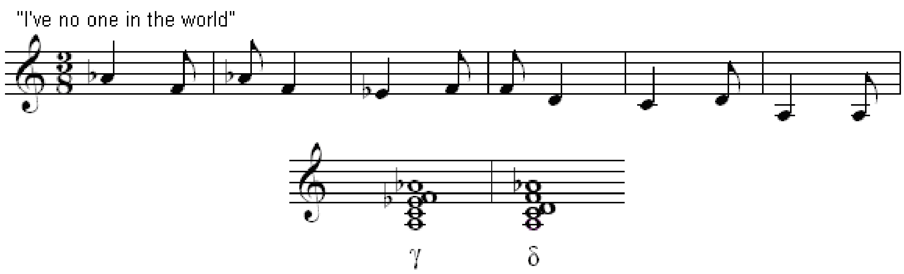

Fig. 4 *Structures of alpha chord*

Another construction model based on the same generator cells with values of 2, 3, 5 and 8 semitones is intended to achieve specific musical structures by joining equal intervals. Thus, by the succession of major seconds (2 + 2 + 2 + 2 + 2 + 2) is built the whole tone scale; by joining minor thirds (3 + 3 + 3 + 3) is obtained the diminished seventh chord, by joining

[4] Ernő Lendvai, *Béla Bartók: An Analysis of his Music,* p. 48

perfect fourths (5 + 5 + 5 ...) is created the forth chords, and by overlapping the minor sixth (8 + 8 + 8) is created the augmented chord.

The composer's preference for these musical intervals, which by their content in semitones corresponds to the numerical value of the elements within the Fibonacci sequence, has become a basic elements of a new sound system in which the musical scales used are built. Each interval becomes the central element of the generating cell of the scale[5]

These sound structures that can fit into the sphere of neo-modal musical scales are:
The 1: 2 model - built from the alternation of a minor second (1) with a major second (2)

C – C# - E b - E – F# - G - A - B b - C.

Model 1: 3 - made by alternating a minor second (1) with a minor third (3):

C – C# – E – F –G# – A – C.

Model 1: 5 - made by alternating a minor second (1) with a perfect fourth (5):

C – C# - F# – G – C.

In addition, all scales have a specific symmetry whose cells produce symmetrical figures visible on the fifth quadrant.

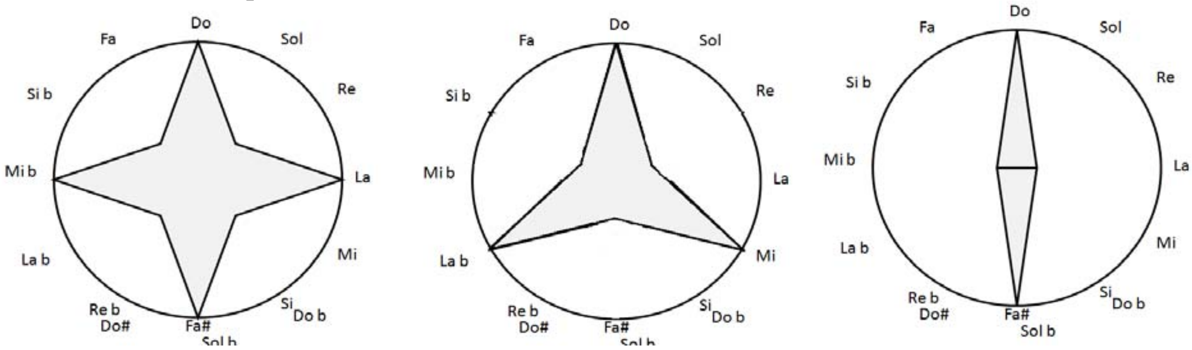

Fig. 5. *Model 1:2; 1:3; 1:5.*

Conclusions

In conclusion, the presence of the golden section in the sound structures can not be neglected. The sound scales corresponding to the divine proportion are found in various hypostases, starting from the modal to the tonal or non-modal system. Thus, can be observed some similarities based on the golden ratio occurring between the pentatonic or heptatonic diatonic modes and the major or minor tonality scale.

An eloquent example is offered by what we might call the most representative type of major chord, which can be imagined as a sound reflection of the golden ratio proportions. This sound representation is contoured by two directions that ultimately conceal the same principle of construction. On the one hand, this chord appears as a selection from the harmonic series whose values correspond to the Fibonacci numbers (1, 2, 3, 5, 8), and on the other hand it is a reflection of a construction principle in which the central element was the quantity of the constituent intervals: 3 st + 5 st = 8 st (major chord in first inversion). As a matter of fact, about the major chord, Matila Ghyka quoting A. Denéréaz in the *Estetică și teoria artei,* argues that the position of the note mi in the inner of the fifth do-sol, it is in golden ratio relation. The second type of construction, however, is a feature of the bartokian

[5] Ernő Lendvai, *Béla Bartók: An Analysis of his Music,* p. 51

language in which the composer applied this pattern to other sound structures in order to discover and use sounds corresponding to the golden section

In conclusion, we can say that besides other ways of representing the golden ratio in nature, architecture, biology, music also has a special place in the interpretation (decryption) of this magical formula. It is visible in the intervals of the elaborated sound structures, either as an expression of a natural selection type, or as a principle of the construction of musical intervals, the basic matter of any kind of sound, harmonic or melodic structure.

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